# BASIC HYDRAULICS 

AN<br>INTRODUCTION TO FIRE STREAM PRACTICES

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## INTRODUCTION

The fire service pump operator (engineer) must supply adequate hose streams with the correct amounts of water at precise pressures in order to insure an effective and safe stream at the nozzle. The engineer must rely upon experience and quick, effective guides to perform this vital function. The engineer is often not allowed sufficient time to perform long hydraulics formulas and exacting slide rule calculations.

The majority of pump operations are performed through a time consuming process of trial and error, which often results in increased fire size, increased radio traffic and equipment damage. As an engineer, you have a limited amount of time to react to the rapid changes that occur within the fire environment.

This workbook is designed to give you the knowledge necessary to adapt to these rapid changes and provide a safe and effective operating environment.

It should be emphasized that the method used for determining proper pump discharge pressure within this text is only one of many possible methods that may be used. The other methods are just as accurate for what they were designed. We feel that this method is more flexible and is better suited for the type of hoselays described in this text.

It is also important to remember that many times theories that work on paper do not always work during "in-field" situations.

The students will be allowed a plus or minus 5 psi. error, as friction loss calculators from different manufacturers vary. This does not excuse the student from computing the problem and demonstrating an acceptable explanation as to why their answer is different from the textbook answer.

This text is designed to help you achieve the correct answer, step by step. It will be your responsibility to interpret all portions and perform rudimentary mathematics correctly. By the end of this text you should be able to determine pump discharge pressure (PDP) quickly and correctly.

## DETERMINING PUMP DISCHARGE PRESSURE (PDP)

Pump discharge pressure is the amount of pressure in pounds per square inch (psi) indicated on the pressure gauge or any given discharge gauge on the apparatus. Stream pattern remaining the same, pump discharge pressure is the major controlling factor that changes nozzle pressure and volume of discharge. Adjusting the pump throttle changes pump discharge pressure and volume output.

PUMP DISCHARGE PRESSURE:
PDP = NP +/- H + FL

## Where:

$$
\left.\left.\begin{array}{rl}
\text { PDP }= & \text { Pump Discharge Pressure } \\
& \text { Pressure at the discharge side of the pump }
\end{array}\right\} \begin{array}{rl}
\mathrm{NP}= & \text { Nozzle Pressure } \\
& \text { Pressure at the nozzle }
\end{array}\right\} \begin{aligned}
+/-\mathrm{H}= & \text { Head (elevation differential) } \\
& \begin{array}{l}
\text {.433psi will lift water vertically } 1 \text { foot; } 43.3 p s i \text { will lift water } \\
\\
\text { vertically } 100 \text { feet. }
\end{array} \\
\mathrm{FL}= & \text { Friction loss per every } 100 \text { feet of hose }
\end{aligned}
$$

We will use the standard for this text of 0.5 psi will lift water vertically 1 foot or 1 psi will lift water vertically 2 feet. This standard will be discussed in greater detail further on in the text.

## PUMP DISCHARGE PRESSURE: (continued)

Elements that you must have as a fire engine operator in order to correctly calculate pump discharge pressure (PDP) are:

1. Length of hose in the hoselay
2. Diameter of hose
3. Volume flow rate
4. Elevation differential (vertical rise or fall) between hoselay and pump
5. Working nozzle pressure

The above five elements are used in all pumping operations to solve for pump discharge pressure. The relationship between these five elements can be expressed mathematically in the following formula:

$$
\mathrm{PDP}=\mathrm{NP}+/-\mathrm{H}+\mathrm{FL}
$$

The next section discusses each element of the formula and their relationship to determining PDP.

NOZZLE PRESSURE (NP): PDP = $\underline{\mathbf{N P}}+/-\mathrm{H}+\mathrm{FL}$
For our purposes, fire streams are divided into two categories:

1. Straight (Solid) stream nozzles (tips)..................................50psi
2. Fog stream nozzles (combination)...................................100psi

Tips and combination nozzles were designed to work most effectively at the abovedesignated pressures.

All fog streams in this workbook are considered to be combination nozzles with the exception of the "Forester" (R5) nozzle, which will always be computed as a straight stream nozzle at 50psi.

A straight stream nozzle is used to supply large concentrations of water in one specific location or in cases where a long reach is required.

Fog stream nozzles are designed to break up the water into small particles or droplets. This increases the surface area of the water increasing heat absorption relative to solid streams.

When computing pump discharge pressure, the proper nozzle pressure is the objective and must be established to calculate the correct pump discharge pressure.

- Remember, all problems in this workbook are calculated with straight stream tips operating at 50 psi and combination nozzles operating at 100 psi.

If a fire engine is pumping low volume ( $<80 \mathrm{gpm}$ ) a short distance ( $<50$ ') through a $11 / 2^{\prime \prime}$ hose to a small fire, with a combination nozzle, we really only need to consider the "NP" element of the pump discharge pressure formula. In this case, the formula would be PDP = NP. But, note that we must always consider the other four elements of the PDP formula. In this case we considered volume, distance, hose diameter and presumed elevation to be zero.

The desired pressure for a combination nozzle is 100psi. By replacing the NP element with 100 psi, we have PDP $=100$ psi.

## HEAD PRESSURE (ELEVATION DIFFERENTIAL): <br> $$
\mathbf{P D P}=\mathbf{N P}+/-\underline{\mathbf{H}}+\mathbf{F L}
$$

Head is also known as lift, back pressure, gravity loss or gain.

## By definition:

Head is the height of water that is necessary to create a given pressure at its base.

Head is measured in terms of vertical feet of water; one foot of head is equivalent to a column of water one foot high. One foot of head exerts a pressure of .433 psi at the base. Two feet of head exerts a pressure of .866psi (2x.433) at the base and so on. Conversely, to raise water one foot you would have to create .433 psi at its base. To raise water two feet you would have to create .866 psi at its base.

The standard rule of thumb used to determine head pressure when solving problems in this workbook will be:

### 0.5 POUNDS PER SQUARE INCH WILL LIFT WATER 1 FOOT 1.0 POUND PER SQUARE INCH WILL LIFT WATER 2 FEET

The reverse is also true:

## - 0.5 POUNDS PER SQUARE INCH WILL BE GAINED FOR EACH 1 FOOT VERTICAL LOSS (drop) IN ELEVATION. <br> - 1.0 POUND PER SQUARE INCH WILL BE GAINED FOR EACH 2 FEET VERTICAL LOSS (drop) IN ELEVATION.

Now, consider that we have a water tank filled to a level 2 feet higher than its base. The pressure exerted by the water at the base of the tank is 1 psi. If we attached a pressure gauge to a water outlet at the base of the tank we could expect to have a water pressure of 1 psi.

## HEAD PRESSURE (continued)

This also means that if we had a water tank filled to a level of 50 feet higher than the base, we can presume that the pressure exerted by the water at the base of the tank will be 25psi ( $50 \mathrm{ft} / 2 \mathrm{psi}$ per ft ). If we attached a pressure gauge to a water outlet at the base of the tank we could expect to have a water pressure of 25 psi.

Fire engine operators must consider the effects of changes in elevation to supply proper water pressure to the hoselay to provide a proper nozzle pressure.

For instance:
A fire engine is pumping a 300 -foot hoselay up a hillside. The vertical change in elevation between the fire engine pump and the top of the hoselay is a 100 feet rise.

## What is the required Pump Discharge Pressure to overcome head pressure (gravity) and force water to the end of the hoselay?

By using our rule of thumb, 0.5 psi will lift water vertically 1 foot then 50 psi will lift water vertically 100 feet $(+\mathrm{H})$. If you engaged the water pump on your fire engine and adjusted the pressure to 50 psi, you can expect water to be forced to the top of the hoselay. (If no other factors influenced the flow of water.) We will discuss those other factors later.

In the previous example we have only solved the head pressure element (+/-H) in the Pump Discharge Pressure formula. If an R-5 nozzle were added to the end of the hoselay, pump discharge pressure would have to be increased to supply an effective water stream. Solving the formula; $\mathrm{PDP}=\mathrm{NP}+/-\mathrm{H}$, the result is $\mathrm{PDP}=50 \mathrm{psi}($ desired NP$)+50 \mathrm{psi}(+\mathrm{H}), \mathrm{PDP}=$ 100psi.

- Remember, vertical changes in elevation are only considered when solving for head pressure. Head pressure is not a function of hoselay length, but rather the gain ( + ) or loss $(-)$ in elevation relative to the pump.

If we had a downhill hoselay with a vertical drop of 100 feet, and a KK type (combination) nozzle attached to the end of the hoselay, what would the desired Pump Discharge Pressure be?

- NOTE: Only solve PDP = NP +/-H.

PDP = 100psi (desired NP) - 50psi (downhill head pressure)
PDP $=100$ psi -50 psi
PDP $=50 \mathrm{psi}$
So, when the pressure is adjusted to read 50psi on the pump control panel, the pressure at the nozzle will be 100 psi. (Presuming all other factors were negligible.)

Downhill hoselays with substantial changes in elevation can be a major problem. The accumulated high water pressure is a safety hazard to the nozzle person and/or any other person in the immediate area. Equipment damage can also be expected. How to overcome these problems will be discussed in greater detail in the classroom.

- Remember, anytime the nozzle is above the pump add the head pressure to the PDP formula; and when the nozzle is below the pump, subtract head pressure from the PDP formula.


## FRICTION LOSS (FL): PDP = NP +/-H + $\underline{\text { FL }}$

The next element in the PDP formula is friction loss (FL). When moving water comes in contact with the inner lining a hose, a loss of energy will occur due to friction. This energy loss is expressed in pounds per square inch loss. This friction results in an eddying of the water or turbulence within the hose. The "drag" that results consumes energy.

Eddying is caused by the difference in the flow rates of water at the center of the hose and at the outer edges against the lining. The water flowing along the outer edge of the hose is slowed down when it comes in contact with the lining due to friction. The water in the center of the hose flows faster because it is not in contact with any surface. These differences create eddies which retards total water flow. As gpm flow or volume is increased, turbulence is increased, increasing total resistance to flow. Eventually, a point is reached where flow reaches a maximum flow per given length for any given hose.

Maximum flow is that volume of water that can be passed through a given length and diameter hose where the psi required to overcome the FL at that flow does not exceed the limitations of any single part of the system (i.e. broken hose, plumbing or pump).

Since hoselays vary in length from fire to fire, it is easiest to consider friction loss (FL) in terms of a common unit. The calculator supplied with this workbook expresses FL per every one hundred feet of hose. This will be explained in greater detail later.

## FACTS ABOUT FRICTION LOSS:

1. The smaller the diameter of the hose, the greater the friction loss.
2. Friction loss factors are computed on 100 -foot sections of hose. For each 100 -foot length of hose added, whether uphill or downhill, the friction loss of each length added must be determined.
3. The larger the diameter of hose, the less friction loss involved.
4. $3 / 4$ inch hardlines create 8 times the friction loss of 1 -inch hose.
5. 1-inch hose creates 6 times the friction loss of $1 \frac{1}{2}$ inch hose.
6. Friction loss increases 4 times, if the volume is doubled through a given size hose.
7. If the cross sectional area of a hose is doubled, FL decreases 4 times.

## REDUCING FRICTION LOSS:

- REDUCE NOZZLE PRESSURE: If the nozzle pressure is reduced, the volume discharged (GPM output) will be less; therefore, the friction loss will be less. This may prevent the fire stream from performing the required task.
- REDUCE NOZZLE SIZE, MAINTAINING SAME NOZZLE PRESSURE: Reducing the nozzle size and maintaining the same nozzle pressure reduces gpm discharged.

CAUTION: The quantity of water being discharged may not be sufficient to cool and completely extinguish the fire.

- LAY PARALLEL HOSE LINES OR INCREASED HOSE DIAMETER: With all other factors remaining constant, two parallel lines will have $1 / 4$ the friction loss of a single line of the same diameter and length, transporting the same quantity of water. Three lines will have $1 / 9$ the friction loss of a single line, and four lines will have $1 / 16$ the friction loss.


## SAFETY WARNING!!

## - ANYTIME YOU MAKE CHANGES WHICH WILL AFFECT A HOSELAY, IMMEDIATELY NOTIFY ALL NOZZLE OPERAOTRS USING THE LAY PRIOR TO THE MODIFICATION!!

FRICTION LOSS IN APPLIANCES: $\operatorname{PDP}=\mathrm{NP}+/-\mathrm{H}+\mathrm{FL}+\underline{\mathbf{A}}$
Appliance friction loss should be ignored when determining friction loss in a wildland hose lay. This is true for any hose-lay length, including very long hose lays with thousands of feet of hose and numerous tees or wye valves, because appliance friction loss is a very small percentage of the total friction loss in a hose lay. Extensive testing by SDTDC has shown that the actual friction loss for appliances such as hose-line tees and gated wye valves is less than 1.0 psi. The pressure losses in $1-1 / 2$ inch appliances at $50 \mathrm{gal} / \mathrm{min}$ were as follows:

- Wye valve
0.46 psi
- Hose line tee
0.05 psi
- Hose line tee w/ valve
0.77 psi

Based on these results, ignoring friction loss in appliances is more accurate than arbitrarily assigning a 5 psi pressure loss to each appliance in a hose lay.

NOTE: the Pump Discharge Pressure formula is now complete:
PDP = NP +/-H + FL

A reasonable estimate of friction loss can be made by using the following equation developed by Lewis F. Moody, Princeton University.
$\mathrm{FL}=\mathrm{C}(\mathrm{Q} / 100)^{2}(\mathrm{~L} / 100)$
Where:
FL= Friction loss
C= friction loss coefficient
$\mathrm{Q}=$ Flow (gallons per minute)
L= Hose lay length (feet)

| Hose <br> size | Friction Loss <br> Coefficient |
| :--- | :--- |
| $5 / 8$ | 2000 |
| $3 / 4$ | 1100 |
| 1 | 250 |
| $11 / 2$ | 35 |
| $13 / 4$ | 14 |

STRAIGHT STREAM TIPS - These are the actual GPM flows for forester tips. Use these in all hydraulic calculations. It is recommended to write the GPM flow on your friction loss calculator next to the tip size for quick reference so you will not have to use the top window.
$\begin{array}{lllll}\text { Tip Orifice Size (in) }- & \frac{3 / 16^{\prime \prime} \text { tip }}{7 G P M} & \frac{1 / 4 " \text { tip }}{13 G P M} & \frac{5 / 16^{\prime \prime} \text { tip }}{21 \mathrm{GPM}} \quad \frac{3 / 8^{\prime \prime} \text { tip }}{30 \mathrm{GPM}} \quad \underline{1 / 2^{\prime \prime} \text { tip }} \\ \text { Flow (gal/min) }- & 73 \mathrm{GPM}\end{array}$

Remember if the hydraulics problem states a Tip size use 50 psi (forester nozzle) If the hydraulics problem states a flow rate use 100 psi (KK or other fog nozzle).

## DETERMINING PDP

## PROBLEM \#1



100 ft . of 1 " line

The illustration shows an engine pumping, on flat ground, a 100-foot section of 1" diameter hose. Attached to the end of the hose is a Forester R-5 nozzle with a $1 / 4$ " tip. What is the correct pump discharge pressure to supply 50psi to the nozzle?

STEP \#1 is to write down the PDP formula.
$+\mathrm{NP}=$
+/-H =

+ FL $=$
PDP
(Remember, R-5 nozzles are considered straight stream nozzles (tips) designed to operate at 50psi).
STEP \#2 is to solve for all knowns in the PDP formula prior to using the calculator. We know NP = 50psi, +/-H = 0 (there is no rise or loss in elevation). Your PDP formula should now look like the one below.
$+\mathrm{NP}=50$
$+/-\mathrm{H}=0$ (flat land)
$+\mathrm{FL}=$ ? ?
PDP ??
The only element left to solve for is Friction Loss (FL).
STEP \#3 using the chart above determine the flow rate for a given tip size ( $1 / 4^{\prime \prime}$ tip= 13 gpm .).
We have determined that 13gpm will be flowing through the 100 -foot section of 1 " hose.
STEP \#4

Determine the amount of friction loss (psi loss) per 100 feet in length that 13gpm produces when flowing through a 1 " hose.
$\mathrm{FL}=\mathrm{C}(\mathrm{Q} / 100)^{2}(\mathrm{~L} / 100)$
$\mathrm{C}=250$ (from the chart above)
$\mathrm{Q}=13$ (13 gpm from chart above)
$\mathrm{L}=100$ (hose length is 100 Ft .)

```
FL=250(13/100)2(100/100)
FL=250(.13)2 (1)
FL=250 *.0169
FL=4.225
Rounded to 4 psi
```



100 ft . of 1 " line $=4 \mathrm{psi}$ loss
As the above illustration shows, we now have all the elements needed to solve for PDP. It is recommended you label any problems you are solving in this method or a similar manner in order to keep track of all the necessary information. On any test this is how you show your work.

STEP \#5 is to fill in FL and determine PDP.
$+\mathrm{NP}=50$
$+/-\mathrm{H}=0$

+ FL = 4
PDP $=54$ psi
The correct answer for our problem is PDP = 54psi. When pump discharge pressure is adjusted to 54psi on the pump panel water is forced through the hoselay. Eddy currents are formed creating a 4 psi loss within the 100 -foot section of hose. 50 psi is the pressure remaining when the water reaches the nozzle. With a $1 / 4^{\prime \prime}$ tip, 13gpm will exit the nozzle and supply the desired fire stream.


## PROBLEM \#2

You are pumping a $11 / 2^{\prime \prime}$ hoselay 500 feet long with a $5 / 16^{\prime \prime}$ tip on level ground. What is your pump discharge pressure??


STEP \#3 Using the chart above a 5/16" tip delivers 21gpm.
STEP \#4 Using the fancy math (like in problem 1) or the handy chart on page 34 Determine the friction loss for a $1 \frac{1 / 2 "}{}$ hose with 21 gpm flowing through it.


## STEP \#5

```
Fill in the FL
value and
solve the PDP
problem.
+NP =50
+/-H = 0 Flat-Flat- Flat (we are in Kansas now)
\(+F L=10\) (2psi FL X 5100 foot sections of hose) (round up)
PDP \(=60 \mathrm{psi}\)
```


## DETERMINING PDP WITH HEAD <br> (ELEVATION DIFFERENTIAL):

PROBLEM \#3 You are pumping 800 feet of 1" hose 50 vertical feet above the pump with a $3 / 8$ " tip. What is the pump discharge pressure?


NOTE: We now have a change in elevation requiring a value for $+/-\mathrm{H}$. The vertical change is 50 feet above the pump so you must add pressure to the PDP formula. If 1psi will raise water vertically 2 feet then 25psi will raise water vertically 50 feet.

STEP \#3 Using the chart on page 8 a $3 / 8$ " tip is 30 gpm .
STEP \#4 If you like long math, have fun. Otherwise use the chart on page 34, a 1" hose will have a friction loss value of 23psi for each 100 foot section of hose.

STEP \#5 Multiply the number of 100 ft sections by the FL per 100-foot section.

| Fill in the FL <br> value and <br> solve the PDP <br> problem. +NP $=50$ <br> $+/-\mathrm{H}$ <br>  $=25$ <br> +FL $=184=(8$ lengths of hose X 23 psi$)$ |  |
| :--- | :--- |
|  | ------------- |

## DETERMINING PDP: (continued)

Before proceeding to the next problem be sure you understand how to correctly complete the previous three. Most mistakes occur by misreading the charts, using the wrong nozzle pressure, forgetting to properly add or subtract the head pressure or omitting the number of 100 foot sections of hose when determining FL in the hoselay.

NOTE: Care must be taken when reading the values. Round up to the nearest whole number when the value is 0.5 to 0.9 . Round down when the value is 0.1 to 0.4 . For example, if you read a value of 2.3, round down to 2.0. If you read a value of 2.7, round up to 3.0 .

PROBLEM \#4 You are pumping 500 feet of $1 \frac{1}{2}$ " hose through a gated wye to two sections of 1 " hose each 100 feet long with $5 / 16$ " tips on flat ground. What is the pump discharge pressure?


Visualize how the water will flow in problem \#4. Pressurized water is forced out of the pump through the 500' section of $11 / 2$ " hose to the gated wye. An equal amount of water is then forced through each lateral. If the nozzle sizes differed the amounts out each line would differ also. The sum of the water flowing thru each nozzle will have to travel through the $1 / 1 / 2$ " line.

SOLVING PDP: $\quad+\mathrm{NP}=50$
$+/-\mathrm{H}=0$
$+\mathrm{FL}=$ ?
$+\mathrm{FL}=$ ?
PDP
Note that the FL element is divided into 1" FL and $11 / 2$ " FL. Resistance to flow must be figured for both lengths of hose.

DETERMINING PDP: (continued)


Problem \#4 now illustrates that the pump must discharge 42gpm to supply fire streams to each nozzle.

SOLVING PDP:

$$
\begin{aligned}
+\mathrm{NP} & =50 \\
+/-\mathrm{H} & =0 \\
+1 " \mathrm{FL} & =11^{*}(1 \mathrm{X} 11) \\
+1 / 1 / 2 \mathrm{FL} & =30 \quad(5 \mathrm{X} 6)
\end{aligned}
$$

$$
\text { PDP = } 91
$$

* NOTE: RULE OF THUMB! When solving FL for hoselays with more than one lateral, only use the FL of the lateral, which will require the greatest PDP to achieve the desired working pressure. We only solve for one nozzle. If we solve for the nozzle requiring the greatest PDP to achieve the desired working pressure than all other nozzles will be supplied adequately if not in excess. In problem \#4 the FL for each lateral is the same so a value of 11 psi is used. This will be discussed in greater detail during the classroom session.

DETERMINING PDP: (continued)

PROBLEM \#5
You are the Engineer on a Model 46 Fire Engine and have been dispatched to a reported smoke in a rural area. Upon arrival you see flames impinging on an occupied structure. Your Captain decides to make an external attack on the structure by deploying 300' of $11 / 2^{\prime \prime}$ hose with a 100 gpm combination nozzle. What should you set your PDP to, to supply a safe and effective fire stream??

## $11 / 2^{\prime \prime}$

100 GPM
$300^{\prime}$

Step \#2 $+\mathrm{NP}=100^{*}$
$+/-\mathrm{H}=0$
$+\mathrm{FL}=$ ??

*REMEMBER: It is common for combination nozzles to operate at 100psi.

STEP \#3:By using the chart on page 35you can see that you will have 35psi FL per each 100 foot section of hose.

STEP \#4: Fill in the proper FL value and complete the PDP problem.

SOLVING PDP:
$+\mathrm{NP}=100$
$+/-\mathrm{H}=0$
$+11 / 2 " F L=105$ (3 X 35)
$P D P=205$

## DETERMINING PDP FOR PROGRESSIVE HOSELAYS:

PROBLEM \#6 Your BLM Heavy Engine ( type 4) and a five-person support crew have been assigned to contain a flare-up in Division C. The Captain decides to deploy three Gansner packs using 20gpm combination nozzles to suppress the flare-up and the many small spot fires starting on the unburned side of the line. As the Engineer, what is the correct pump discharge pressure?
(Note: Each gansner pack consists of: a 100' section of $1 \frac{1}{2}$ " hose; a $1 \frac{1}{2}$ " gated wye with a $11 / 2 "$ to 1 " reducer; a 100 section of 1 " hose; and a $1^{\prime \prime}$ combination nozzle.)


Before solving the problem, visualize how the water will flow through the hoselay and exit the nozzles ( see below).


In order to supply each nozzle with 20 gpm , the pump must discharge 60 gpm which flows through the first 100 ' section of $11 / 2^{\prime \prime}$ hose to gated wye \#1. At gated wye \#1 20gpm flows through the first lateral and exits the nozzle. 40gpm flows through the second section of $11 / 2 "$ hose to gated wye \#2. At gated wye \#2 20gpm flows through the second lateral and exits the nozzle. The remaining 20 gpm flows through the third section of $11 / 2$ " hose to gated wye \#3 and exits through the lateral.

The path requiring the greatest PDP to achieve the desired working pressure is from the pump through all three sections of $11 / 2$ " hose and through lateral \#3. When solving for PDP you must compute the FL for each 100 ' section of $1 \frac{1}{2 \prime \prime}$ hose and lateral \#3.

SOLVING PDP:

$$
\begin{aligned}
& +\mathrm{NP}=100 \\
& +/-\mathrm{H}=0 \\
& +1 \text { "FL = ? } \\
& +11 / 2 \mathrm{\prime FL}=\text { ? } \\
& +11 / 2^{\prime \prime} \text { FL = ? } \\
& +11 / 2^{\prime \prime} \mathrm{FL}=\text { ? } \\
& \mathrm{PDP}=\text { ??? }
\end{aligned}
$$

$$
\begin{aligned}
& +\mathrm{NP}=100 \\
& + \text { /-H = } 0 \\
& +1 \text { "FL = } 10 \\
& +11 / 2^{\prime \prime} \text { FL }=1 \\
& +11 / 2 " \text { FL }=6 \\
& +1 \frac{1}{2} \text { "FL }=13 \\
& \operatorname{PDP}=130
\end{aligned}
$$

It is recommended you set up the PDP formula in the manner shown above. You will be less likely to make a mistake when computing PDP, especially when there are several sections of hose with various amounts of flow.

Remember to solve for FL for only one lateral: the lateral that will require the greatest PDP to achieve the desired working pressure. In problem \#6 all the laterals are the same length (100') and diameter (1") and have identical nozzles.

When the water being discharged by the pump is pressurized to 130psi each nozzle will deliver the desired fire stream for safe and effective application.

## PROBLEM \#7:

You have been assigned as a portable pump operator on a fire. A road on one flank and a handline that surrounds many fingers on the other flank contains the fire. The I.C. has decided to burn out the handline and begin mop-up prior to strong winds predicted for the next operational period. A 1,000-gallon porta-tank with a Mark III pump is located at the top of the fire where the handline and road meet. The hoselay extends from the port-tank downhill along the handline to the road. The vertical drop between the pump and the bottom of the fire is 100 feet. What is the PDP needed to supply 100psi to the bottom nozzle?


SOLVING PDP:

$$
\begin{aligned}
& +\mathrm{NP}=100 \\
& +1 \text { " FL = } 10 \\
& +11 / 2 " \mathrm{FL}=2 * \\
& +11 / 2^{\prime \prime} \mathrm{FL}=12 * \\
& +11 / 2^{\prime \prime} \mathrm{FL}=26 \text { * } \\
& \text { PDP }=150 \\
& -\mathrm{H}=50^{* *}
\end{aligned}
$$

PDP $=100$
*These amounts are double the values indicated on the calculator because there are two 100 ' sections of $1 \frac{1}{2}$ " hose between each gated wye.
** The +/-H element of the PDP formula was moved underneath PDP as a reminder to subtract the head pressure because the nozzle is lower in elevation than the engine.

NOTE: Problem \#6 \& \#7 used a $11 / 2 "$ diameter hose to supply water to the 1 " laterals. In the wildland fire environment you may hear the term supply line or trunk line describing the main water supply line for laterals.

If you have been assigned to pump a hoselay already in place, don't assume the main supply line or trunkline to be a $11 / 2^{\prime \prime}$ hose. (i.e. it could be a 1 " line supplying $3 / 4$ " laterals.) Verify the hoselay!

## PROBLEM \#8:

You are the Engineer of a Model 52 fire engine and part of a task force consisting of a dozer, 4,000-gallon water tender, and two Model 20 fire engines assigned to a 50 -acre brush fire. Upon arrival, the IC instructs your engine to initiate a hoselay up the right flank of the fire and hold and begin mop-up operations. Figure \#19 shows the completed hoselay. What is the correct pump discharge pressure?

Figure \#19:

SOLVING PDP:
$+\mathrm{NP}=100$
$+\mathrm{H}=150$

$$
\begin{aligned}
& \text { +1" FL = } 10 \text { (1 X 10) } \\
& +1 \frac{1}{2}>\text { FL }=2 \text { (2 X } 1 \text { ) } \\
& +11 / 2 \text { "FL }=12 \text { (2 X 6) } \\
& +11 / 2 \text { " } \mathrm{FL}=26 \text { ( } 2 \mathrm{X} 13 \text { ) } \\
& +11 / 2 \mathrm{FL}=44 \text { (2 X 22) } \\
& \text { PDP = } 344
\end{aligned}
$$



You may realize that the Model 52 fire engine is incapable of pumping the hoselay in Problem \#8, as it is configured. Alternatives to overcome this problem will be discussed in the classroom.

It was previously stated, one method to reduce excessive friction loss is the use of parallel hoselays. The following examples (Problem \#9 \& \#10) show how FL is reduced when this parallel hoselays are used.

## DETERMINING FL IN PARALLEL HOSELAYS

PROBLEM \#9: You are pumping the hoselay shown below consisting of a 1500' $1 \frac{1 / 2 "}{2}$ trunk line supplying two $3 / 8^{\prime \prime}$ tips. What is the PDP?

FIGURE \#20:


SOLVING PDP:

$$
\begin{aligned}
&+\mathrm{NP}= 50 \\
&+/-\mathrm{H}= 0 \\
&+1 " \mathrm{FL}=23(1 \mathrm{X} 23) \\
&+11 / 2 " \mathrm{FL}=3(1 \times 3) \text { Friction Loss in 100' with 30gpm flow. } \\
&+11 / 2^{\prime \prime} \mathrm{FL}=195(15 \mathrm{X} 13) \text { Friction Loss in } 1500 \text { with 60gpm flow. } \\
&----------------- \\
& \text { PDP }=271
\end{aligned}
$$

FIGURE \#21 below illustrates the use of parallel lines to reduce FL. Two $11 / 2$ " trunk lines 1500 ' in length are supplying the laterals. A gated wye is used directly off the pump discharge and a Siamese and gated wye are coupled together at the first lateral.


1500" 1 1/2" hose, 30 GPM

The pump must discharge 60gpm in order to supply the proper flow for each nozzle, Figure \#21 shows the flow distribution that will take in a parallel hoselay. 30gpm will flow through each parallel line merging at the Siamese. 30gpm is then discharged by the first lateral and the remaining 30 gpm continues through the last lateral.

- When solving for FL in a parallel lay you only consider the loss in one side of the parallel lines. The pressure in each line will be the same so if your PDP overcomes the FL in one, your PDP will overcome the FL in both. Therefore the FL in only one line is of any concern.

SOLVING PDP:/

$$
\begin{aligned}
& +\mathrm{NP}=50 \\
& +/-\mathrm{H}=0 \\
& +1 \text { " FL = } 23 \text { Friction Loss in 100' with 30gpm flow. } \\
& +11 / 2 " \text { FL }=3 \text { Friction Loss in 100' with 30gpm flow. } \\
& +11 / 2 " \text { FL }=45 \text { Friction Loss in 1500' with 30gpm flow. } \\
& \text { PDP }=121
\end{aligned}
$$

When comparing the conventional hoselay in Problem \#9 and the parallel hoselay in Problem \#10 we find: There is a sizable difference in the required pump discharge pressure; the parallel lay required approximately twice the amount of hose as the single lay; having available resources and time to install a parallel lay could be a problem; the FL in the 1500' of $1 \frac{1}{2}$ " hose in the conventional lay has 4 times the FL as the parallel lay.

Try solving problem \#8 (page 19) with 200 feet of parallel hose running off of the back of the model 52. Can the model 52 pump this lay now? Try 200 feet off of the engine being parallel and the 200 feet between the $1^{\text {st }}$ and $2^{\text {nd }}$ laterals being parallel. What could you do to the hoselay to accomplish your mission?

## SUMMARY

This text has shown the steps for calculating hydraulics problems you may encounter in the wildland fire environment. Following are sample problems with answers designed to increase your efficiency as an Engineer. The problems should be completed prior to attending the Academy. With practice, you should be able to complete these problems in a short period of time.

## ACKNOWLEDGEMENTS

Andy Parker and Chuck Whitlock, Greenville Ranger District, Plumas National Forest, 1978, originally developed this text.

Revised 1983.
Revised 1992 by Mark Levitoff, Almanor Ranger District, Lassen National Forest.
Revised 1998 by Ralph C. Schurwanz, Doublehead Ranger District, Modoc National Forest.

Revised 2000 by Julie Zoppetti, Doublehead Ranger District, Modoc National Forest.
Revised 2004 by Phillip Shafer, Mt Hough Ranger District, Plumas National Forest.
Revised 2006 by Jim Burton, Coconino National Forest and Bob Travis, Prescott N.F.
Revised 2007 by Jim Burton \& Keith Halloran, Coconino National Forest.
Revised 2009 by Clay W. Fowler GJD-BLM, White River NF, GVRD-GMUG NF, CNMNPS.

## IMPORTANT INSTRUCTIONS FOR SAMPLE AND QUIZ PROBLEMS!!!!

- \#1. Sample Problem \#1 shows 250' of 3/4" hardline. This is equivalent to $2^{1 ⁄ 2} 100$ ' sections of hose when computing FL.
- \#2. It is recommended you solve the easiest problems first, and then proceed to the others.
- \#3. Bring your workbook, regular calculator, and sample problems to the Academy.
- \#4. Only the sample problems include an answer sheet.


## SAMPLE PROBLEMS (1-15)



$$
\mathrm{PDP}=
$$

$\qquad$
2.


$$
\mathrm{PDP}=
$$



PDP = $\qquad$
4.


$$
\mathrm{PDP}=
$$

$\qquad$
5.


PDP = $\qquad$


PDP = $\qquad$
7.


PDP =
8.

9.


## PDP =

$\qquad$
10.

11.


PDP = $\qquad$
12.


PDP =


$$
\mathrm{PDP}=
$$

$\qquad$
14.


PDP =
15.


## ANSWERS TO SAMPLE QUESTIONS

1. $\mathrm{PDP}=235$

| 20 GPM |  |  |
| :---: | :---: | :---: |
| NP= 100psi |  |  |
| H= 25psi | FL | GPM |
| $\frac{\text { FL }}{3 / 4 "=110=2.5 \times 44}$ | 44 | 20 |
| PDP $=235 p s i$ |  |  |

2. $\quad \mathrm{PDP}=97$

| 5/16" nozzle@50psi=21gpm |  |  |
| :---: | :---: | :---: |
| $\mathrm{NP}=\mathbf{5 0 p s i}$ | FL | GPM |
| FL 1"= $11=1 \times 11$ | 11 | 21 |
| FL $11 / 2>=36=6 \times 6$ | 6 | 42 |
| PDP $=97 \mathrm{psi}$ |  |  |

3. $\quad \mathrm{PDP}=59$

1/4" nozzle@50psi=13gpm
$\mathrm{NP}=50 \mathrm{psi}$
$\mathrm{H}=-10 \mathrm{psi}$
FL 1" $=4=1 \times 4 \quad 4 \quad 13$
FL $11 / 2 "=1=1 \times 1 \quad 1 \quad 13$
FL $11 / 2 "=14=7 x 2 \quad 26$ PDP = 59psi
4. $\quad \mathrm{PDP}=142$

| 90 GPM |  |  |
| :---: | :---: | :---: |
| NP $=100 \mathrm{psi}$ |  |  |
| H $=0$ 0psi | FL | GPM |
| $\frac{\text { FL } 11 / 2 "=42=1.5 \times 28}{}$ | 28 | 90 |
| PDP $=142 \mathrm{psi}$ |  |  |

5. $\quad \mathrm{PDP}=130$

| 20 GPM |  |  |
| :---: | :---: | :---: |
| NP=100psi |  |  |
| H= 0psi | FL | GPM |
| FL $1 "=10=1 \times 10$ | 10 | 20 |
| FL $11 / 2 "=1=1 \times 1$ | 1 | 20 |
| FL $11 / 2 "=1 \times 1 \times 6$ | 6 | 40 |
| FL $11 / 2 "=13=1 \times 13$ | 13 | 60 |
| PDP $=130 p s i$ |  |  |

6. $\quad \mathrm{PDP}=282$

7. $\quad \mathrm{PDP}=223$

3/8" nozzle@50psi=30gpm
$\mathrm{NP}=50 \mathrm{psi}$
$\mathrm{H}=60 \mathrm{psi}$
FL GPM
FL 1"= 23=1x23 $23 \quad 30$
FL $11 / 2 "=91=7 \times 13 \quad 1360$
PDP = 224psi
8. $\quad \mathrm{PDP}=207$

9. $\quad \mathrm{PDP}=260$

| 3/8" nozzle@50psi=30gpm |  |  |
| :---: | :---: | :---: |
| NP= 50psi |  |  |
| $\mathrm{H}=110 \mathrm{psi}$ | FL | GPM |
| FL 1" $=23=1 \times 23$ | 23 | 30 |
| FL 11/2"= 3=1x3 | 3 | 30 |
| FL $11 / 2$ " $=26=2 \times 13$ | 13 | 60 |
| FL $11 / 2$ = $=48=16 \times 3$ | 3 | 60(30) |
| PDP $=260 \mathrm{psi}$ |  |  |

10. $\quad \mathrm{PDP}=162$

| $1 / 4 "$ nozzle@50psi=13gpm |  |  |
| :---: | :---: | :---: |
| NP= 50psi |  |  |
| H= 60psi | FL | GPM |
| FL $1 "=4=1 \times 4$ | 4 | 13 |
| FL $11 / 2 "=1=1 \times 1$ | 1 | 13 |
| FL $11 / 2 "=2=1 \times 2$ | 2 | 26 |
| FL $11 / 2 "=45=9 \times 5$ | 5 | 39 |
| PDP $=162 p s i$ |  |  |

11. $\mathrm{PDP}=133$

| 80 GPM |  |  |
| :---: | :---: | :---: |
| NP=100psi |  |  |
| H=0psi | FL | GPM |
| $\frac{\text { FL } 11 / 2 "=33=1.5 \times 22}{}$ | 22 | 80 |
| PDP $=133 p s i$ |  |  |
|  |  |  |

12. $\quad \mathrm{PDP}=297$

| 20 GPM |  |  |
| :---: | :---: | :---: |
| NP=100psi |  |  |
| H=100psi | FL | GPM |
| FL $1 "=10=1 \times 10$ | 10 | 20 |
| FL $11 / 2 "=1=1 \times 1$ | 1 | 20 |
| FL $11 / 2 "=6=1 \times 6$ | 6 | 40 |
| FL $11 / 2 "=26=2 \times 13$ | 13 | 60 |
| FL $11 / 2=54=18 \times 3$ | 3 | $60(30)$ |
| PDP= 297 psi |  |  |
|  |  |  |

13. $\quad \mathrm{PDP}=118$

| $3 / 8 "$ nozzle@50psi=30gpm |  |  |
| :---: | :---: | :---: |
| NP $=$ 50psi |  |  |
| H= 30psi | FL | GPM |
| FL 1"= 23= 1x23 | 23 | 30 |
| FL 11/2" $=15=5 \times 3$ | 3 | 30 |
| PDP $=118 p s i$ |  |  |

14. $\quad \mathrm{PDP}=165$

3/8" nozzle@50psi=30gpm
$\mathrm{NP}=50 \mathrm{psi}$
$\mathrm{H}=0 \mathrm{psi} \quad$ FL $\quad$ GPM
FL 1" $=115=5 \times 23 \quad 230$
PDP= 165psi
15. $\mathrm{PDP}=220$ to the $3 / 8$ " nozzle.

| 1/4" tip @50psi =13gpm |
| :---: |
| 3/8" tip@50psi=30gpm |
|  |
| NP= 50psi |
| H= 70psi |
| FL $1 "=23=1 \times 23$ |
| $\frac{\text { FL }}{}$ |
| FL 112 GPM $=77=7 \times 11$ |
| PDP= 220psi |

3/8" tip@50psi=30gpm
$\mathrm{NP}=50 \mathrm{psi}$
$\mathrm{H}=70 \mathrm{psi} \quad$ FL GPM
FL 1"= 23=1x23 2330
FL 11/2"= 77=7x11 1156
PDP $=220 \mathrm{psi}$

|  |  |  |  |  |  |  | 5/8 | 3/4 | 1 | $11 / 2$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Hose Size |  |  |  | 53 | 561.8 | 308.99 | 70.225 | 9.8315 |
|  | GPM | 5/8 | 3/4 | 1 | $11 / 2$ | 54 | 583.2 | 320.76 | 72.9 | 10.206 |
|  | 1 | 0.2 | 0.11 | 0.025 | 0.0035 | 55 | 605 | 332.75 | 75.625 | 10.5875 |
|  | 2 | 0.8 | 0.44 | 0.1 | 0.014 | 56 | 627.2 | 344.96 | 78.4 | 10.976 |
|  | 3 | 1.8 | 0.99 | 0.225 | 0.0315 | 57 | 649.8 | 357.39 | 81.225 | 11.3715 |
|  | 4 | 3.2 | 1.76 | 0.4 | 0.056 | 58 | 672.8 | 370.04 | 84.1 | 11.774 |
|  | 5 | 5 | 2.75 | 0.625 | 0.0875 | 59 | 696.2 | 382.91 | 87.025 | 12.1835 |
|  | 6 | 7.2 | 3.96 | 0.9 | 0.126 | 60 | 720 | 396 | 90 | 12.6 |
|  | 7 | 9.8 | 5.39 | 1.225 | 0.1715 | 61 | 744.2 | 409.31 | 93.025 | 13.0235 |
|  | 8 | 12.8 | 7.04 | 1.6 | 0.224 | 62 | 768.8 | 422.84 | 96.1 | 13.454 |
|  | 9 | 16.2 | 8.91 | 2.025 | 0.2835 | 63 | 793.8 | 436.59 | 99.225 | 13.8915 |
|  | 10 | 20 | 11 | 2.5 | 0.35 | 64 | 819.2 | 450.56 | 102.4 | 14.336 |
|  | 11 | 24.2 | 13.31 | 3.025 | 0.4235 | 65 | 845 | 464.75 | 105.625 | 14.7875 |
|  | 12 | 28.8 | 15.84 | 3.6 | 0.504 | 66 | 871.2 | 479.16 | 108.9 | 15.246 |
|  | 13 | 33.8 | 18.59 | 4.225 | 0.5915 | 67 | 897.8 | 493.79 | 112.225 | 15.7115 |
|  | 14 | 39.2 | 21.56 | 4.9 | 0.686 | 68 | 924.8 | 508.64 | 115.6 | 16.184 |
|  | 15 | 45 | 24.75 | 5.625 | 0.7875 | 69 | 952.2 | 523.71 | 119.025 | 16.6635 |
|  | 16 | 51.2 | 28.16 | 6.4 | 0.896 | 70 | 980 | 539 | 122.5 | 17.15 |
|  | 17 | 57.8 | 31.79 | 7.225 | 1.0115 | 71 | 1008.2 | 554.51 | 126.025 | 17.6435 |
|  | 18 | 64.8 | 35.64 | 8.1 | 1.134 | 72 | 1036.8 | 570.24 | 129.6 | 18.144 |
|  | 19 | 72.2 | 39.71 | 9.025 | 1.2635 | 73 | 1065.8 | 586.19 | 133.225 | 18.6515 |
|  | 20 | 80 | 44 | 10 | 1.4 | 74 | 1095.2 | 602.36 | 136.9 | 19.166 |
|  | 21 | 88.2 | 48.51 | 11.025 | 1.5435 | 75 | 1125 | 618.75 | 140.625 | 19.6875 |
|  | 22 | 96.8 | 53.24 | 12.1 | 1.694 | 76 | 1155.2 | 635.36 | 144.4 | 20.216 |
|  | 23 | 105.8 | 58.19 | 13.225 | 1.8515 | 77 | 1185.8 | 652.19 | 148.225 | 20.7515 |
|  | 24 | 115.2 | 63.36 | 14.4 | 2.016 | 78 | 1216.8 | 669.24 | 152.1 | 21.294 |
|  | 25 | 125 | 68.75 | 15.625 | 2.1875 | 79 | 1248.2 | 686.51 | 156.025 | 21.8435 |
|  | 26 | 135.2 | 74.36 | 16.9 | 2.366 | 80 | 1280 | 704 | 160 | 22.4 |
|  | 27 | 145.8 | 80.19 | 18.225 | 2.5515 | 81 | 1312.2 | 721.71 | 164.025 | 22.9635 |
|  | 28 | 156.8 | 86.24 | 19.6 | 2.744 | 82 | 1344.8 | 739.64 | 168.1 | 23.534 |
|  | 29 | 168.2 | 92.51 | 21.025 | 2.9435 | 83 | 1377.8 | 757.79 | 172.225 | 24.1115 |
|  | 30 | 180 | 99 | 22.5 | 3.15 | 84 | 1411.2 | 776.16 | 176.4 | 24.696 |
|  | 31 | 192.2 | 105.71 | 24.025 | 3.3635 | 85 | 1445 | 794.75 | 180.625 | 25.2875 |
|  | 32 | 204.8 | 112.64 | 25.6 | 3.584 | 86 | 1479.2 | 813.56 | 184.9 | 25.886 |
|  | 33 | 217.8 | 119.79 | 27.225 | 3.8115 | 87 | 1513.8 | 832.59 | 189.225 | 26.4915 |
|  | 34 | 231.2 | 127.16 | 28.9 | 4.046 | 88 | 1548.8 | 851.84 | 193.6 | 27.104 |
|  | 35 | 245 | 134.75 | 30.625 | 4.2875 | 89 | 1584.2 | 871.31 | 198.025 | 27.7235 |
|  | 36 | 259.2 | 142.56 | 32.4 | 4.536 | 90 | 1620 | 891 | 202.5 | 28.35 |
|  | 37 | 273.8 | 150.59 | 34.225 | 4.7915 | 91 | 1656.2 | 910.91 | 207.025 | 28.9835 |
|  | 38 | 288.8 | 158.84 | 36.1 | 5.054 | 92 | 1692.8 | 931.04 | 211.6 | 29.624 |
|  | 39 | 304.2 | 167.31 | 38.025 | 5.3235 | 93 | 1729.8 | 951.39 | 216.225 | 30.2715 |
|  | 40 | 320 | 176 | 40 | 5.6 | 94 | 1767.2 | 971.96 | 220.9 | 30.926 |
|  | 41 | 336.2 | 184.91 | 42.025 | 5.8835 | 95 | 1805 | 992.75 | 225.625 | 31.5875 |
|  | 42 | 352.8 | 194.04 | 44.1 | 6.174 | 96 | 1843.2 | 1013.76 | 230.4 | 32.256 |
|  | 43 | 369.8 | 203.39 | 46.225 | 6.4715 | 97 | 1881.8 | 1034.99 | 235.225 | 32.9315 |
|  | 44 | 387.2 | 212.96 | 48.4 | 6.776 | 98 | 1920.8 | 1056.44 | 240.1 | 33.614 |
|  | 45 | 405 | 222.75 | 50.625 | 7.0875 | 99 | 1960.2 | 1078.11 | 245.025 | 34.3035 |
|  | 46 | 423.2 | 232.76 | 52.9 | 7.406 | 100 | 2000 | 1100 | 250 | 35 |
|  | 47 | 441.8 | 242.99 | 55.225 | 7.7315 | 101 | 2040.2 | 1122.11 | 255.025 | 35.7035 |
|  | 48 | 460.8 | 253.44 | 57.6 | 8.064 | 102 | 2080.8 | 1144.44 | 260.1 | 36.414 |
|  | 49 | 480.2 | 264.11 | 60.025 | 8.4035 | 103 | 2121.8 | 1166.99 | 265.225 | 37.1315 |
|  | 50 | 500 | 275 | 62.5 | 8.75 | 104 | 2163.2 | 1189.76 | 270.4 | 37.856 |
|  | 51 | 520.2 | 286.11 | 65.025 | 9.1035 | 105 | 2205 | 1212.75 | 275.625 | 38.5875 |
|  | 52 | 540.8 | 297.44 | $67.6{ }^{34}$ | 9.464 | 106 | 2247.2 | 1235.96 | 280.9 | 39.326 |

## QUIZ PROBLEMS (1-10) <br> NEED TO COMPLETE AND BE RETURNED BEFORE DUE DATE

1. 



PDP $=$ $\qquad$
2.


PDP = $\qquad$


PDP = $\qquad$
4.


PDP =
5.


$$
\mathrm{PDP}=
$$

6. 



PDP =
7.


PDP = $\qquad$
8.


PDP = $\qquad$

10.


PDP =

